The Wikipedia article on weak measurements existing at this time (Nov. 24, 2010) seems to me highly misleading and in some ways simply wrong. I posted a criticism of the article on the "Discussion" page a few days ago, but I am hesitant to attempt to rewrite the article for fear of being drawn into a time-consuming controversy. Were I to rewrite it, it would go something like the following.

If anyone thinks it has sufficient merit to form the basis for a Wikipedia article, I have no objection so long as it is not attributed to me unless presented exactly as I wrote it. I have no doubt that it could be improved, but I am also concerned that I might not agree with some well-meaning "improvements".

Sample encyclopedia article on quantum weak measurement

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"Weak measurement" is a technique to measure the average value of a quantum observable A without appreciably affecting the initial state s of the system being measured. Weak measurements differ from normal (sometimes called "strong" or "von Neumann") measurements in two ways:

1. If A has discrete spectrum (which we assume for simplicity), a strong measurement when the system is in state s yields an eigenvalue of A; if the measurement is repeated many times (starting each time with the system in state s) one obtains a sequence of eigenvalues of A which when averaged yield an approximation to $\langle s|A|s\rangle$, the expectation of A in the state s.

   By contrast, a *weak measurement* only yields a sequence of numbers which *average* to $\langle s|A|s\rangle$. For example, a strong measurement of the spin of a spin-1/2 particle must yield spin 1/2 or -1/2, but a particular weak measurement could yield spin 100 [1], while a subsequent weak measurement on an identical system might be -128.3 . Typically, a single weak measurement gives little information; only the average of a large number of such measurements is meaningful.

2. A strong measurement changes ("projects") an initial pure state s to an eigenvector of A. (The particular eigenvector obtained cannot be predicted, though its probability is determined.) This substantially changes the state s unless s happened to be close to that eigenvector.

   However, a weak measurement does not substantially change the initial state.

Weak measurements are usually implemented by coupling the original system S to be measured with an auxiliary quantum "meter system" M. The meter system is often visualized as a macroscopic meter with a pointer that moves along a scale, though in practice various microscopic quantum systems are used.

The composite system is mathematically represented as the tensor product of S with M, denoted $S \otimes M$. A "product" state in this tensor product is typically denoted $|s\rangle|m\rangle$, where s is a state of S and m a state of M. States which are not product states are called *entangled* states. An example of a state which can be shown to be entangled is

$$e := \sqrt{p}\,|s_1\rangle|m_1\rangle + \sqrt{1-p}\,|s_2\rangle|m_2\rangle,$$

with s_1, s_2 linearly independent states of S with $|s_1| =!s_2| = 1$, m_1, m_2 orthonormal states of M, and $0 < p < 1$ a probability. The state of S corresponding to e is not a "pure" state, but a mixture of pure states, (described by the density matrix $\text{trace}_M P_e$, where trace_M denotes the partial trace over M and P_e the projector to the subspace spanned by e). It is the mixed state which is in pure state s_1 with probability p and in pure state s_2 with probability $1-p$. If p is close to 0 or 1, then one could say that e is "slightly entangled", in which case the corresponding state of S is close to s_2 or s_1, respectively. (If s_1 and s_2 are not independent, then e is not entangled, and the corresponding state of s is s_1.)

Weak measurements of an observable A when the system S is in pure state s_1 can be implemented by putting the composite system in a state like e with p close to 1 and measuring an observable B in the meter system which has eigenvectors m_1, m_2. The measurement will be "weak", in the sense that the state of S after the measurement will differ little from s_1. By suitably choosing the observable B, and the state s_2, it can be arranged that the average value of B when the composite system is in the state e equals (to arbitrary accuracy) the average value of A when S is in state s_1.

To get an idea of how this can work, let ϵ be a small positive parameter and $B = B(\epsilon)$ an observable on M with $Bm_1 = \frac{1}{2\epsilon}m_1$ and $Bm_2 = -\frac{1}{2\epsilon}m_2$. The corresponding observable on $S \otimes M$, which will also be denoted B for notational simplicity, takes a product state $|s\rangle|m\rangle \in S \otimes M$ to $|s\rangle B|m\rangle$.

For s a state of S for which we want to weakly measure $\langle s|A|s\rangle$, define a state $f = f(\epsilon)$ of the composite system as the normalization of $(|s\rangle + \epsilon A|s\rangle)|m_1\rangle + (s - \epsilon A|s\rangle)|m_2\rangle$, namely

$$f = f(\epsilon) := \frac{(|s\rangle + \epsilon A|s\rangle)|m_1\rangle + (|s\rangle - \epsilon A|s\rangle)|m_2\rangle}{|(|s\rangle + \epsilon A|s\rangle)|m_1\rangle + (|s\rangle - \epsilon A|s\rangle)|m_2\rangle|} \quad .$$

Then

$$\langle f(\epsilon)|B(\epsilon)|f(\epsilon)\rangle = \langle s|A|s\rangle + O(\epsilon) \quad ,$$

so that $\langle f|B|f\rangle$ approximates $\langle s|A|s\rangle$ to arbitrary accuracy.

To summarize, the average value of $B(\epsilon)$ in the state $f(\epsilon)$ equals (in the limit $\epsilon \to 0$) the average value of A in the state s. Moreover, the measurements are "weak" in the sense that the state of s after the measurement of B is close to s—the post-measurement state is a mixture of the normalizations of pure states

$s + \epsilon As$ and $s - \epsilon As$, both of which are close to s for small ϵ. The parameter ϵ is a measure of the "weakness" of the measurement: the smaller ϵ is, the less the initial state s is disturbed by the measurement.

This is not the only weak measurement scheme. The seminal paper [1] which introduced weak measurements used a meter space identical to the Hilbert space for a free particle in one dimension, with the "meter" observable B the usual position observable for the free particle[1] so that the "meter" can be visualized as like a pointer moving along a scale. Much of the literature on weak measurements implicitly or explicitly assumes the procedure of [1].

All of the above is uncontroversial, but many aspects of weak measurements *are* controversial. In the literature, weak measurements are often presented in a context in which S is initially in a state s_I (called a *preselected* state) and after the measurement is in a state s_F (called a *postselected* state). One starts with the initial state s_I, performs an "intermediate" measurement of B, and then performs a final measurement to determine whether the state of S is s_F (otherwise it is a mixture of pure states orthogonal to s_F). If the final state is s_F, the result of the intermediate measurement is recorded, otherwise it is discarded. The average of all the recorded measurements of B is called a *weak value* of A.

Since the average of measurements of B *without* postselection equals (in the limits of a large number of measurements with vanishing strength) the average of strong measurements of A, one might be tempted to think that the same would be true *with* postselection. But that overlooks that an intermediate strong measurement of A will project the state of S to an eigenstate of A, which will affect the postselection. Indeed, calculation reveals that the postselected average of strong measurements of A can be no larger than the largest eigenvalue of A (and no smaller than the smallest). Other detailed calculations reveal that the average of measurements of B can turn out to be larger than the largest eigenvalue of A. So, it is controversial whether the postselected average of B may reasonably be considered as a meaningful substitute for any sort of postselected average of A.

Another problem is that different weak measurement procedures can give different postselected meter averages (i.e., averages of B, called "weak values" of A); in fact, in many situations one can obtain arbitrary weak values for A [7] [6]. The seminal paper [1] obtained the weak value

$$\frac{\langle s_F|A|s_I\rangle}{\langle s_F|s_I\rangle} \quad . \tag{1}$$

However, the expression (1) need not be real, and many subsequent authors replace this with its real part.

The motivation of the weak value (1) in [1] is mathematically complicated and its mathematics and physics has been questioned by several authors [3] [4] [2] [5] on different grounds. Moreover, the critics do not always agree with each

[1] Actually, they used the momentum observable for the free particle as their meter observable, but Fourier transformation transforms their picture into the one we describe.

other, which is a further indication of the controversial nature of the weak value concept. Many authors use (1) or its real part as if it were the only possible weak value.

References

[1] Y. Aharonov, D. Z. Albert, and L. Vaidman, "How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100", Phys. Rev. Lett. **60**, 1351-1354 (1988)

[2] I. M. Duck, P. M. Stevenson, and E. C. G. Sudarshan, "The sense in which a 'weak measurement' of a spin-1/2 particle's spin component yields a value 100", Phys. Rev. D **40**, 2112-2117 (1989)

[3] A. Peres, "Quantum measurements with postselection", Phys. Rev. Lett. **62**, 3226 (1989)

[4] A. J. Leggett, "Comment on 'How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100", Phys. Rev. Lett. bf 62, 3225 (1989

[5] S. Parrott, "What do quantum 'weak' measurements actually measure?", www.arXiv.org/abs/0908.0035

[6] S. Parrott, "Quantum weak values are not unique. What do they actually measure?", www.arXiv.org/0909.0295

[7] R. Jozsa, "Complex weak values in quantum measurement", Phys. Rev. A **76**, 044103 (2007)