Review by Stephen Parrott of
Quantum Paradoxes: Quantum Theory for the Perplexed
by
Y. Aharonov and D. Rohrlich

I obtained this book through interlibrary loan, with the intention of buying it if it looked as if it would repay careful study. I decided against buying it for reasons described below.

The loan period was only ten days, and I was only able to read the first eight chapters (about half the book) in this time. However, I did look over the rest, and its flavor seemed typical of the chapters that I did read. Most of the authors' arguments are exceedingly vague.

The preface begins:

> "*Quantum Paradoxes* is a series of studies in quantum theory. Each chapter begins with a paradox motivating the study ... of a fundamental aspect of the theory. ... The studies, taken together, set out a new interpretation of quantum theory."

Before continuing, I should remark that the precise content of the "new interpretation of quantum theory" escaped me. Indeed, I wouldn't have guessed that the book contained such an interpretation had it not been for the above quote. Perhaps the "new interpretation" is implicitly contained in the chapters which I didn't have time to read carefully.

The authors seem to use the word "paradox" in the sense of "any unexpected result of a calculation" [my interpretation, not a quote from the book]. Most of their "paradox"-yielding calculations are of the hand-waving variety. None of them seemed to me to justify the term "paradox".

For example, the Aharonov-Bohm effect demonstrates that a magnetic field can have an observable effect on electrons even when the field is confined to a region which the electrons never enter. However, the observable effect is a statistical quantum effect (an interference pattern), not an effect which would make sense for a single electron within the conceptual framework of classical electrodynamics. The Aharonov-Bohm effect is surely a striking effect unanticipated by classical electrodynamics, but it seems a stretch to call it a "paradox".

Chapter 2 is largely devoted to a description of a thought experiment devised by Einstein to convince Bohr that the time-energy uncertainty relation (see below) need not hold. The authors report that after considerable effort, Bohr resolved this "paradox". But the time-energy uncertainty relation is not a fundamental part of the logical structure of quantum mechanics—it is a kind of assumed generalization of the Heisenberg uncertainty relations (which *are* rigorous consequences of most axiomatizations of quantum mechanics such as those of Von Neumann and Mackey). Why should it be considered a "paradox" if the time-energy uncertainty relation did not hold (as seems quite conceivable to me)?

In Chapter 2, the reader is never advised that there is a fundamental logical difference between the time-energy uncertainty relation and the position-momentum Heisenberg uncertainty relations. The authors do finally recognize this in an extended discussion in Chapter 8. It begins with the following quote from Section 8.1, p. 106.

> "Numerous books and papers claim that a measurement of energy cannot take an arbitrarily short time. They interpret the energy-time uncertainty relation [equation (2.10), $\Delta E \Delta T \geq h$] as follows: the faster the energy measurement, the more uncertain the result. Let us examine some arguments for this interpretation.
>
> i) A simple argument starts with Einstein's equation relating energy and frequency [equation (2.3), $E = h\nu$]. Suppose a quantum wave takes a time T to pass through a measuring device. Since the wave lasts a time T, its Fourier transform is large for frequencies in a range that includes $0 \leq \nu \leq 1/T$. Then $\Delta E = h\Delta\nu \geq h/T$. ii) ..."

If you find this clear and convincing, then you may get more out of the book than I did. To see the difficulties, try to formulate the author's assertions as a rigorous theorem for a wave function $\psi = \psi(x, t)$ satisfying the Schroedinger equation. and then try to prove it. None of my formulations were trivial, either mathematically or physically. Proofs, if any exist, seem probably difficult. (No proof, or reference to a proof, is given in the book.)

Chapter 7 develops a "model for the measurement of an observable" which is used throughout the rest of the book. It attributes this model to von Neumann, but if it is in fact due to von Neumann, it must be in some different form, because the book's development of it is largely mathematical fantasy. An appendix below gives details.

The above makes clear that I think the book has serious flaws. But I would hesitate to say that it is totally without merit for the following reasons.

Aharonov apparently used reasoning of a style similar to the text to suggest the possibility of the striking Aharonov-Bohm effect. (I do not know the details of the history of this effect). The text's motivation of this effect seems to me not sufficiently convincing to bet on the effect before it was observed. Nevertheless, it *was* observed, and it was Aharonov-type thinking which lead to its observation. There may be something to be learned from this.

It may be that the accepted logical structure of quantum mechanics will eventually be recognized as too limited. Perhaps it will be enlarged to encompass and make rigorous the hand-waving kind of arguments presented in this book.

This is the sort of book which I might recommend for purchase by a well-stocked university library with excess funds for acquisitions. Though I did not buy it because my best judgment is that its careful study would be unlikely to repay the effort involved, it is a book which I might like to have available for browsing.

It seems well edited and professionally produced, with good diagrams. For example, its description of the Bohr-Einstein controversy over the time-energy uncertaintly relations includes an elaborate diagram of an antique clockwork mechanism to measure the time of an energy emission. I enjoyed reading about this even though it seems to me (with the hindsight of subsequent development of the logical structure of quantum mechanics by von Neumann and others) an irrelevant historical curiosity.

1 Appendix

This appendix points out a serious mathematical flaw in one of the authors' arguments, in order to give readers a feel for the book's hand-waving style.

In their development of "a model ... for the measurement of an observable" on p. 97, Sec. 7.2, they consider a hypothetical Hermitian operator which they call P_d, but which I'll call it P for simplicity. No special properties other than Hermiticity seem to be assumed for this operator P (except that it is supposed to be "independent" of another operator A_s, whatever that means). (In particular, P is *not* assumed to be the usual momentum operator). They continue as follows (in which I omit irrelevant subscripts like the above d and choose units in which $\hbar = 1$ for notational simplicity):

> "Because P is a quantum operator, some other quantum operator does not commute with it. Let the operator Q be conjugate to P:
>
> $$[Q, P] = i."$$

Note the unexplained and unjustified logical jump from the assertion that there exists an operator which does not commute with P to the much stronger assertion that there exists Q with $[Q, P] = i$. This is typical of the books' exposition.

The above quote seems to assert that for every Hermitian operator P (on a complex Hilbert space of dimension at least 2), there exists a Hermitian operator Q with $[Q, P] := QP - PQ = i$. But this assertion is algebraically incorrect, as the following formal calculation shows. The calculation is mathematically rigorous for bounded operators P, Q (in which case it shows that $[Q, P] = i$ is impossible for bounded P and Q).

Expanding e^{iQt} (t real) in a power series and noting that repeated use of $[Q, P] = i$ implies $[Q^n, P] = inQ^{n-1}$, we have

$$\begin{aligned} e^{itQ}P &= \sum_{n=0}^{\infty} \frac{i^n t^n}{n!} Q^n P \\ &= \sum_{n=0}^{\infty} \frac{i^n t^n}{n!} P Q^n + \sum_{n=0}^{\infty} \frac{i^n t^n}{n!} inQ^{n-1} \\ &= Pe^{itQ} - te^{itQ} \quad . \end{aligned}$$

whence

$$e^{itQ}Pe^{-itQ} = P - t \quad .$$

Since e^{iQt} is unitary, this implies that the spectral measure of P is invariant under translation. In particular, the spectrum of P is the whole real line. So, for most operators P (e.g, if P has countable discrete spectrum), the required Q cannot exist.

Perhaps the authors meant to *assume* that P is of a special form such that Q does exist. But if this is what they intended, they should have clearly said so, instead of obtaining Q via a nonexistent theorem. Forcing readers to sort such things out and guess at the authors' meanings puts an inordinate burden on the readers. Unfortunately, the whole book is like that (as, to be fair, are too many physics texts).

Finally, I should mention that there are other serious logical problems with the authors' development of the "model ... for the measurement of an observablei", but to explain them I would have to reproduce most of Section 7.2. The Preface's claim that "students can use the book even during a first course in quantum mechanics" seems wildly optimistic.