Dear Dr. Chiribella,

I have abandoned my plan to post comments on CDP10 and CDP11 on my website. I don't understand it well enough, and perhaps never will. I have sent a correction concerning the notation of CDP11 to sci.physics.research and sci.math.research. The one on s.p.r appeared several weeks ago, and you can read it there if you are interested. The one on s.m.r never appeared (and was never rejected); I will repost it.

I was hoping to read CDP11 carefully at least through the derivation of the qubit, but I have become stuck at Lemma 30, p. 15. Until I can resolve this, I it seems unproductive to continue because this lemma is used in several critical places.

No doubt many of my questions about Lemma 30 can be simply answered, and the answers will probably make me feel silly. However, I will go through it the proof in detail in the hope that it may give you some understanding of how difficult it is to read CDP in detail (more because of its intricacy than expositional flaws). Perhaps that might be helpful if you ever decide to write this up in a more pedagogical form. I think you may have to do that if you want to have it carefully read.

The questions begin at equation (12). This does follow from Lemma 12 applied to the preparation test $\{p_\phi \phi, (1-p_\phi)\sigma\}$ with σ as in equation (11), but it would be helpful to explicitly say that. However, I don't see why b in equation (12) must be atomic. From Lemma 12 it follows that if ϕ is *not* pure, b is *not* atomic, but equation (12) seems to require the converse: that if ϕ *is* pure, then b *is* atomic. I imagine that the needed fact will appear immediate once one sees the proper way to look at it, but the need to hunt for "proper" ways slows the reader down.

There is a typo in equation (14). On the left side, the probability q should multiply ψ.

Turning to Lemma 12, its proof is not given in CDP11, but is referred to Lemma 8 in CDP10. However, I can't see how the statement of that Lemma 8, which gives an alternate characterization of the operational norm, has any relevance to Lemma 12. Perhaps the Lemma 8 reference is a typo with some other lemma meant.

> Speaking of the operational norm, I might mention that I have also some uncertainty about its definition, which I will only briefly describe. The motivation for the definition of the operational norm of states given on p. 7 of CDP10 seems to me to not quite correspond to the final definition in equation (30), which is also the definition used in CDP11. The difference is that the motivation seems to require that the sup's and inf's in equation (30) be taken over all observation tests $\{a_0, a_1\}$ not *separately* over all effects a_0 and a_1. This leaves doubt in the reader's mind as to which was actually meant, and necessitates reading some later passages twice, once for each guess at the meaning.

> The difficulty of reading the paper increases exponentially with the number of such ambiguities. (My working assumption is that (30) is the actual definition.)
>
> Also, the definition of operational norm for effects given in CDP11 is not the standard definition of "dual norm" for a dual vector space (for the latter, $\rho \in \mathrm{St(A)}$ should be replaced by $\rho \in \mathrm{St}_R\mathrm{(A)}$. A careful reader will wonder if you are using an unusual norm, or if your norm can be shown to be equivalent to the usual dual norm, or what. This is worth at least a remark.

Returning to the proof of Lemma 30 of CDP11, let us pass to the unnumbered equation just below equation (15), which I will call (*). Below this equation, is stated: "by lemma 29, one has $p' \leq p_{max}$". The only way I see to draw this conclusion from Lemma 29 would be if (*) somehow implies that $|\chi)_A = p'\phi' + (1-p')\sigma$ for some normalized pure state σ, but I can't think why this should hold. Since $|\chi)_A$ is completely mixed, we can write

$$|\chi)_A = p''\phi' + (1-p'')\sigma$$

for some p'' and σ, but I don't see why p'' should necessarily equal p'. Again, I may be merely missing the right way to look at it. On the other hand, I also have to consider the possibility that this could be a crucial error.

Another point about which I am unsure is the offhand remark at the bottom of p. 8 of CDP11 that assuming that $D_{AB} = D_A D_B$, every state ρ of AB can be written as

$$\rho = \sum_{i=1}^{D_A} \sum_{j=1}^{D_B} \rho_{ij} \alpha_i \otimes \beta_j \quad , \qquad (**)$$

where $\{\alpha_i\}$ and $\{\beta_j\}$ are bases (presumably *any* bases), respectively, for $\mathrm{St}_R\mathrm{(A)}$ and $\mathrm{St}_R\mathrm{(B)}$. Relation (**) is subsequently used at various critical places such as Lemma 26 (via reference to the 80-year old paper [1]), but if the hypothesis $D_{AB} = D_A D_B$ is ever proved, I haven't been able to find it.

Best wishes,

Stephen Parrott